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Canonical quantization of anisotropic Bianchi I cosmology from scalar vector tensor Brans Dicke gravity
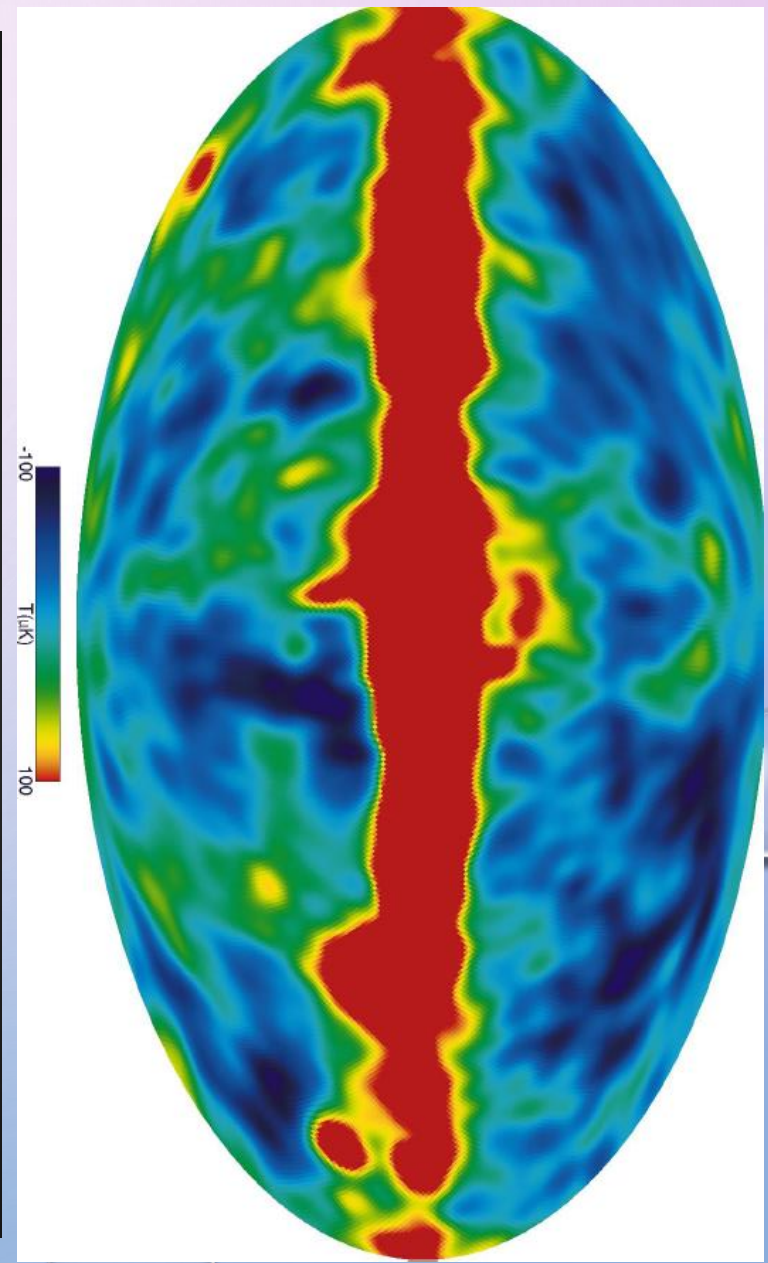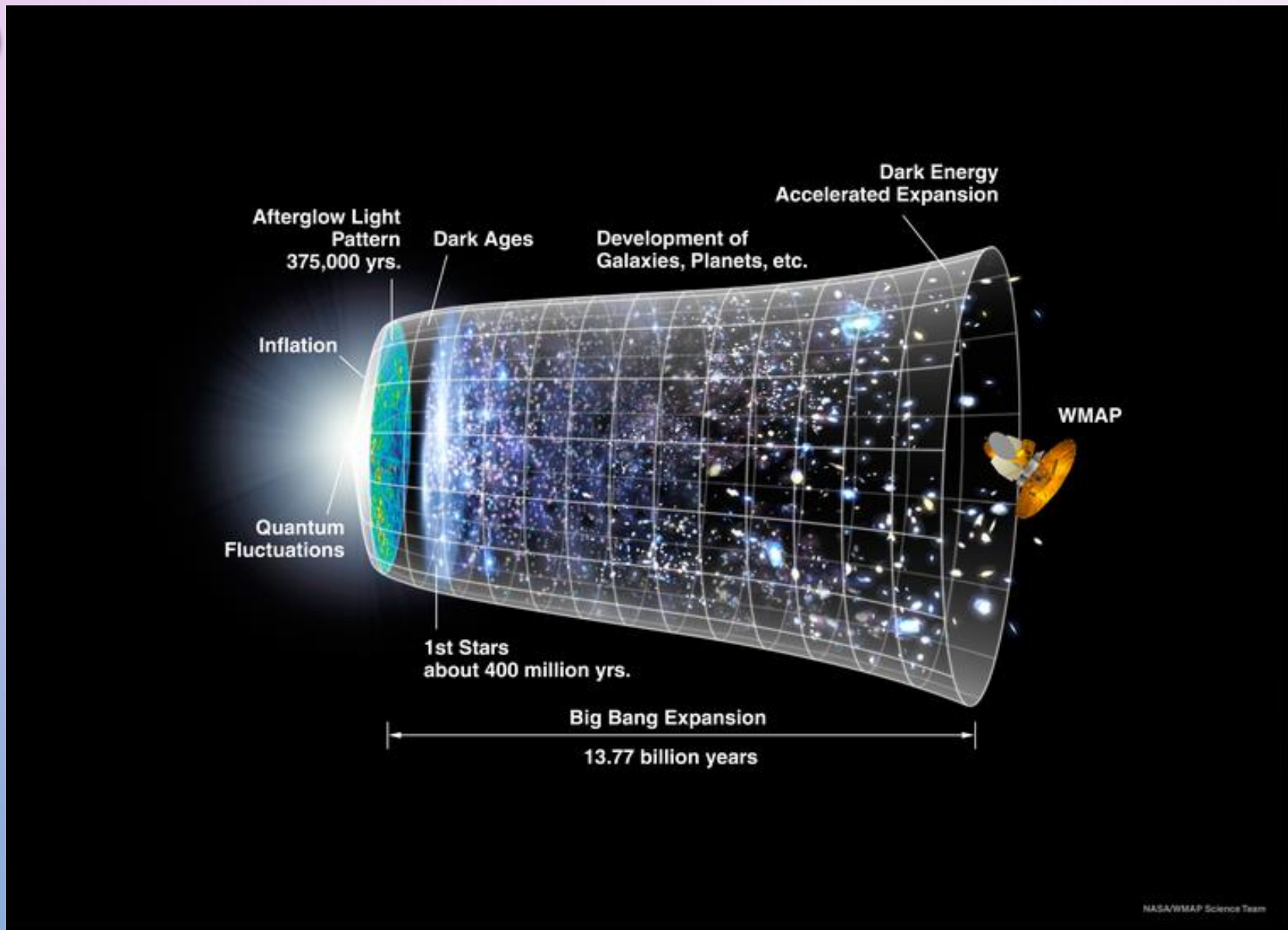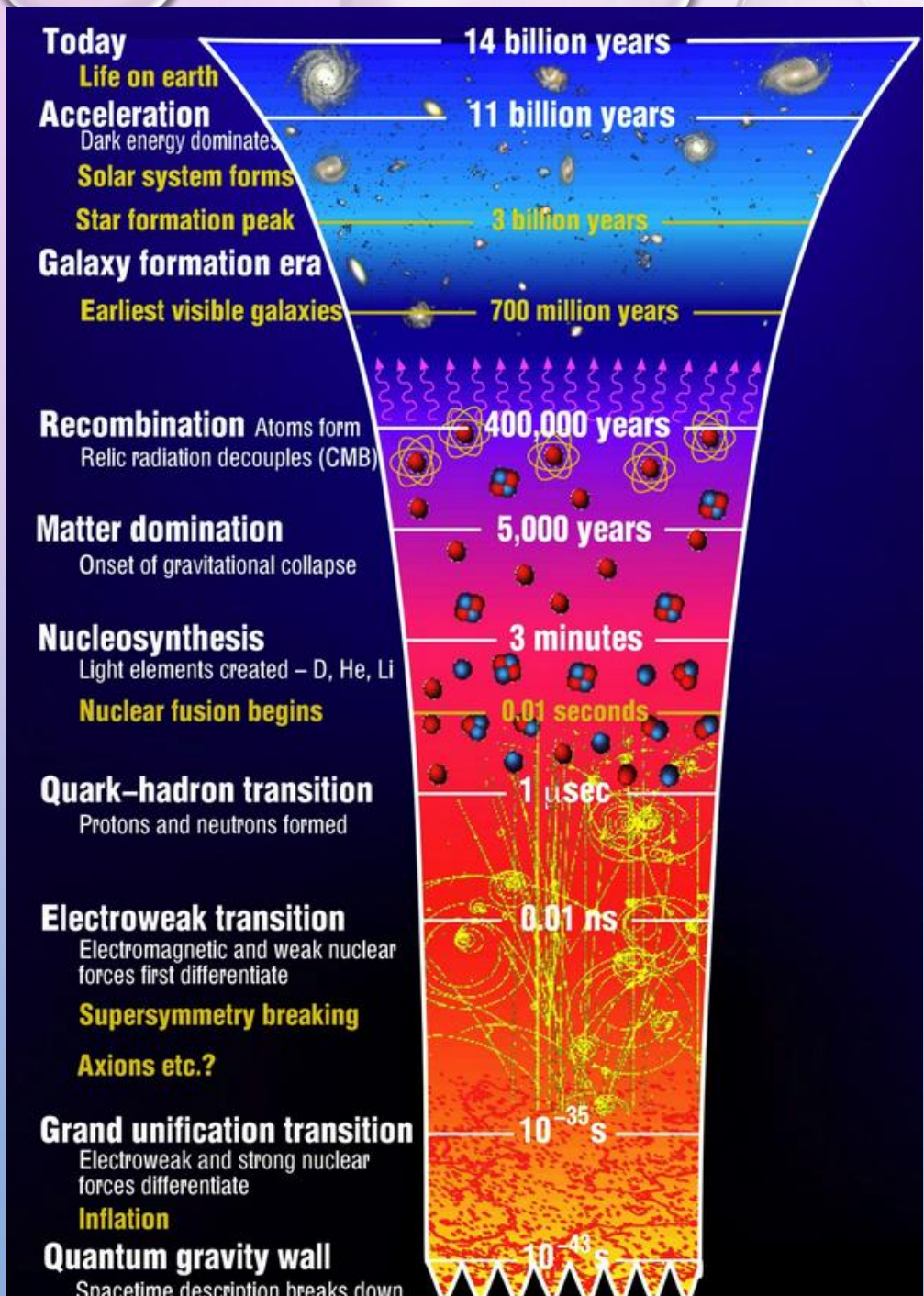
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Abstract. We applied a generalized scalar-vector-tensor Brans Dicke gravity model to study canonical quantization of an anisotropic Bianchi I cosmological model. Regarding an anisotropic Harmonic Oscillator potential we show that the corresponding Wheeler de Witt wave functional of the system is described by Hermit polynomials. We obtained a quantization condition on the ADM mass of the cosmological system which raises versus the quantum numbers of the Hermit polynomials. Our calculations show that the inflationary expansion of the universe can be originate from the big bang with no naked singularity due to the uncertainty principle.
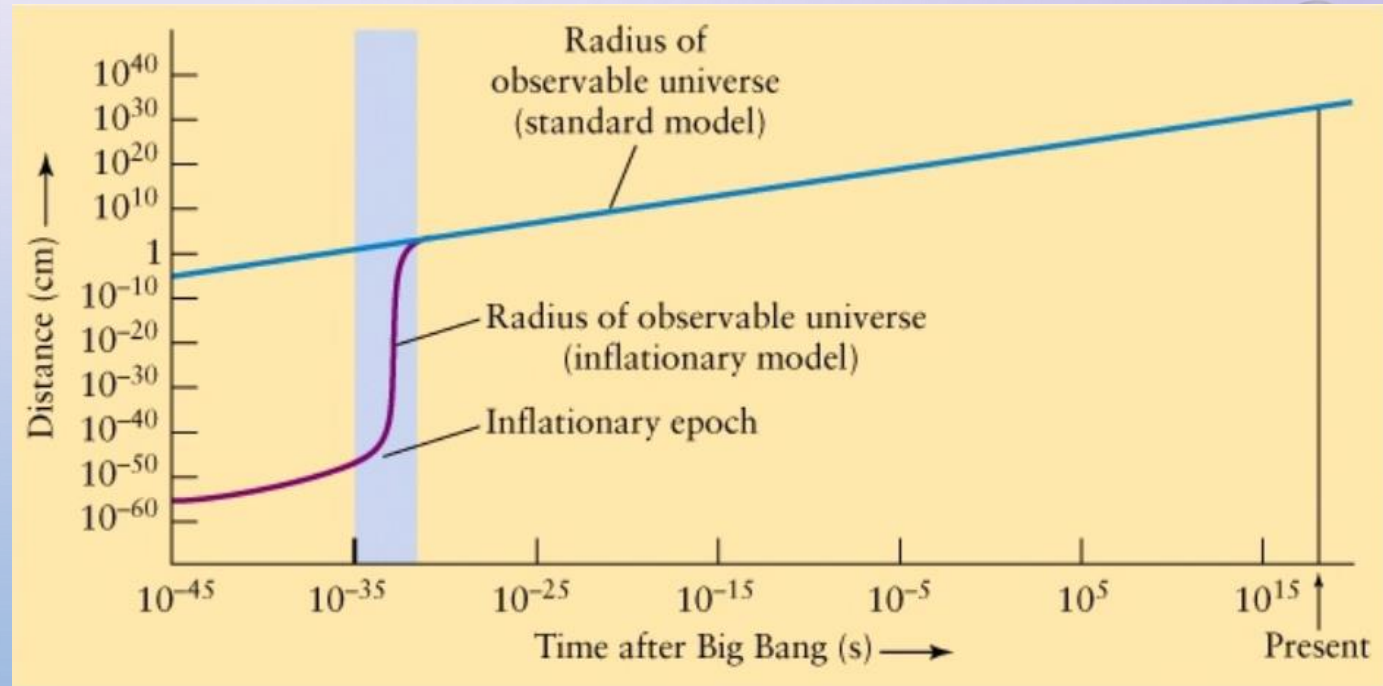
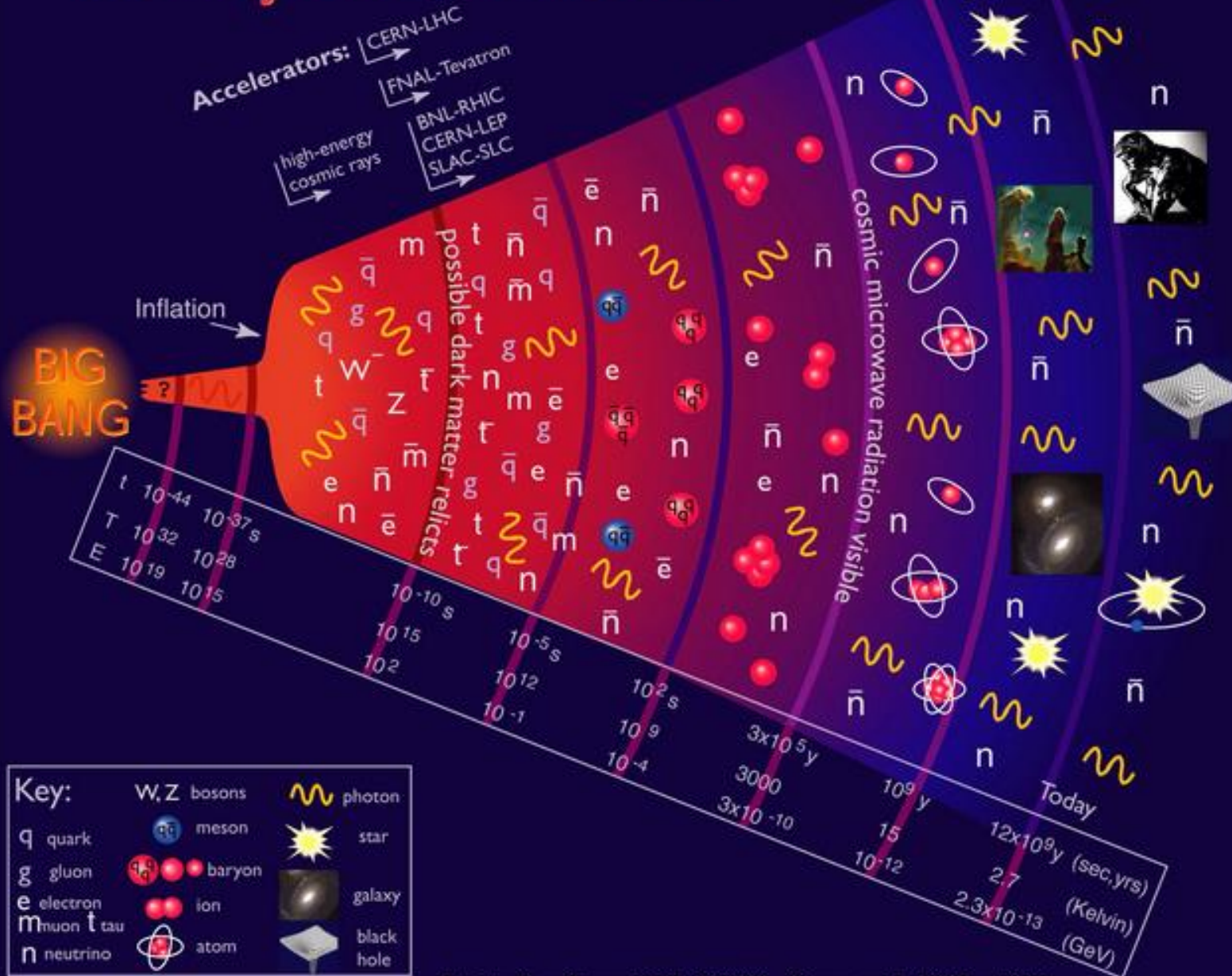# Wilkinson Microwave Anisotropy Probe (WMAP)

Today
Life on earth — 14 billion years
Acceleration — 11 billion years
Dark energy dominates
Solar system forms
Star formation peak — 3 billion years
Galaxy formation era
Earliest visible galaxies — 700 million years

Recombination Atoms form — 400,000 years
Relic radiation decouples (CMB)

Matter domination — 5,000 years
Onset of gravitational collapse

Nucleosynthesis — 3 minutes
Light elements created – D, He, Li
Nuclear fusion begins — 0.01 seconds

Quark–hadron transition — 1 μsec
Protons and neutrons formed

Electroweak transition — 0.01 ns
Electromagnetic and weak nuclear forces first differentiate
Supersymmetry breaking
Axions etc.?

Grand unification transition — 10^{-35} s
Electroweak and strong nuclear forces differentiate
Inflation

Quantum gravity wall — 10^{-43} s
Spacetime description breaks down

George Gamow

Russian-American physicist and science writer
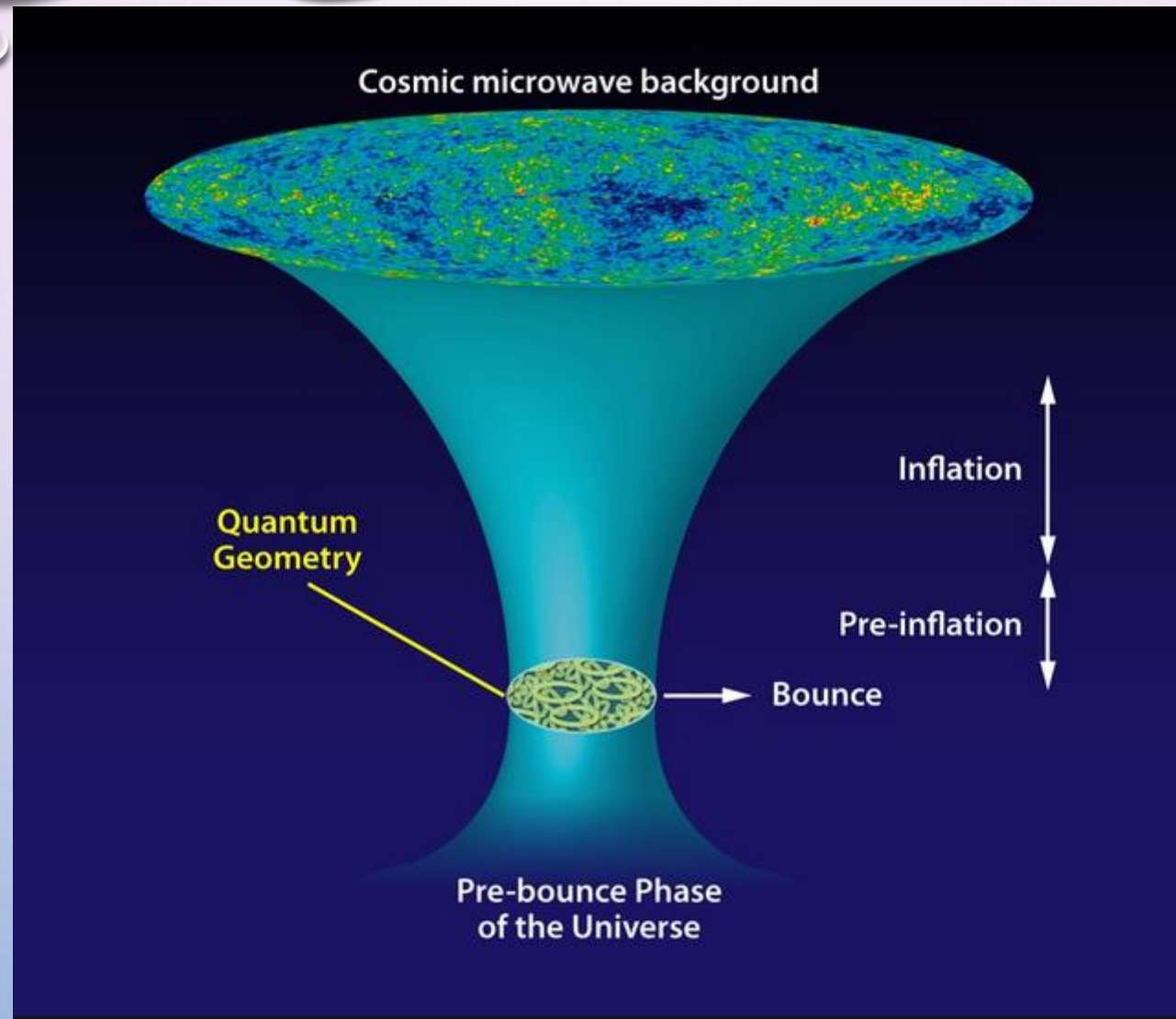
Radius of observable universe (standard model)

Radius of observable universe (inflationary model)

Inflationary epoch

Distance (cm): 10^{40}, 10^{30}, 10^{20}, 10^{10}, 1, 10^{-10}, 10^{-20}, 10^{-30}, 10^{-40}, 10^{-50}, 10^{-60}

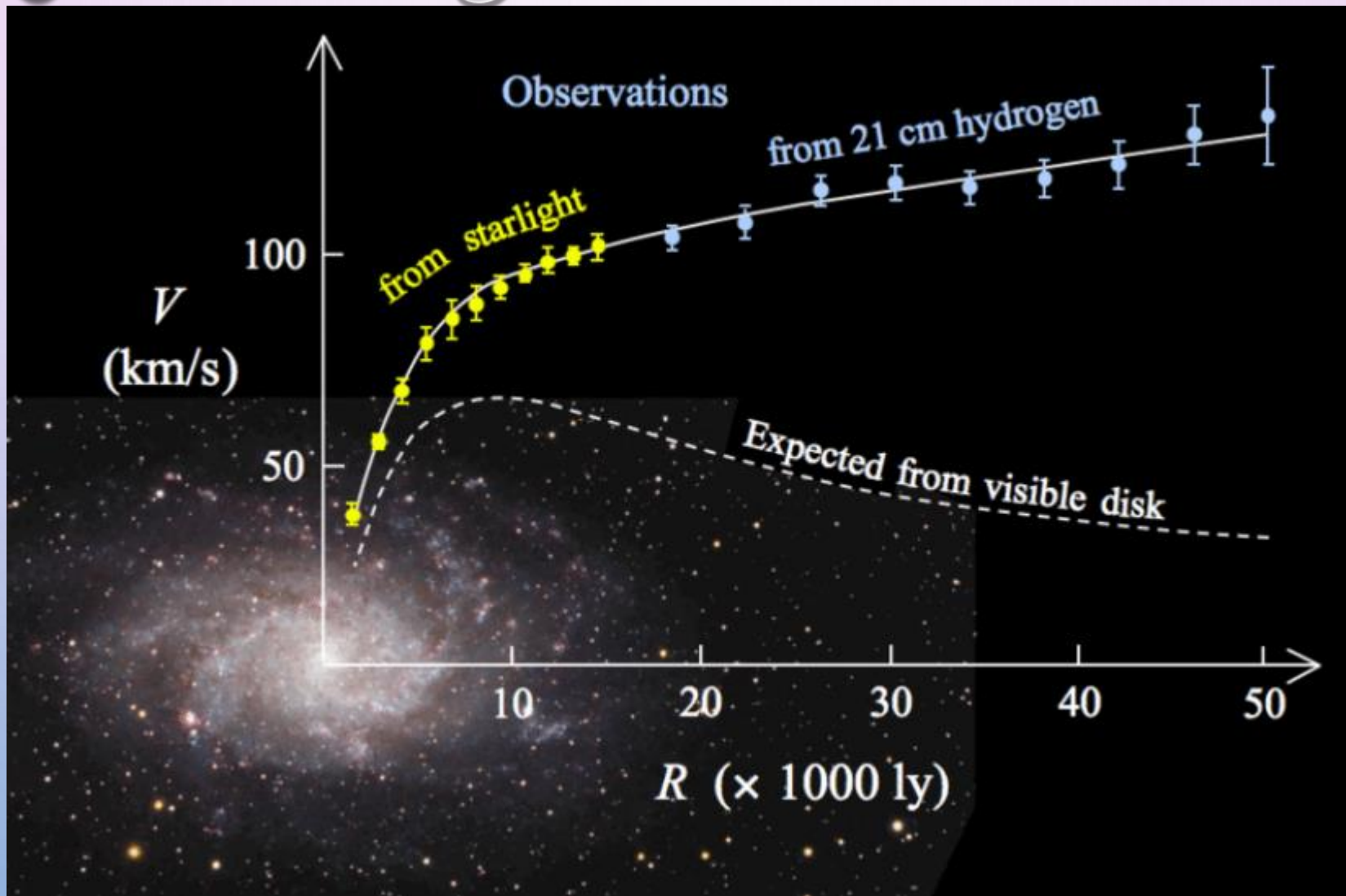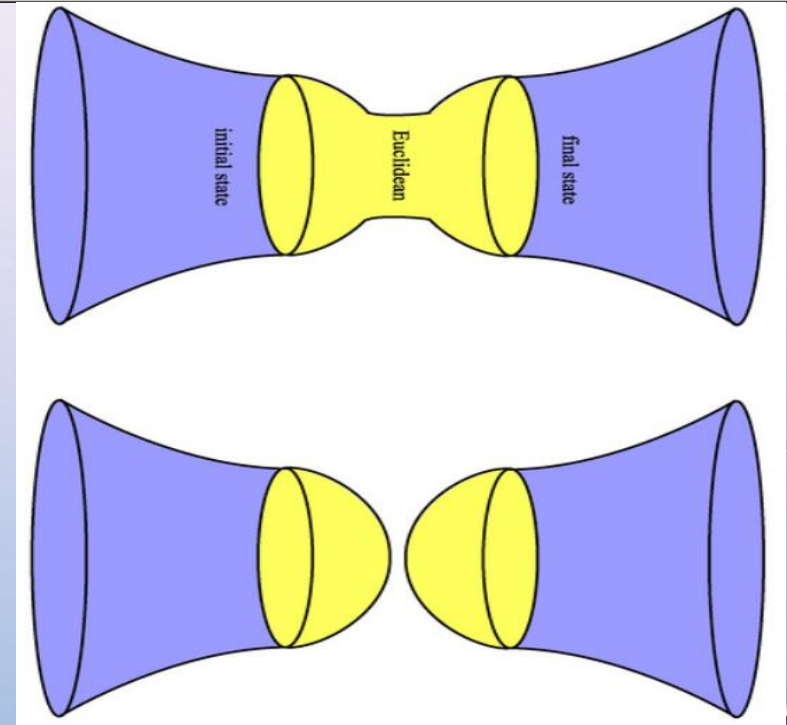Time after Big Bang (s): 10^{-45}, 10^{-35}, 10^{-25}, 10^{-15}, 10^{-5}, 10^{5}, 10^{15} — Present
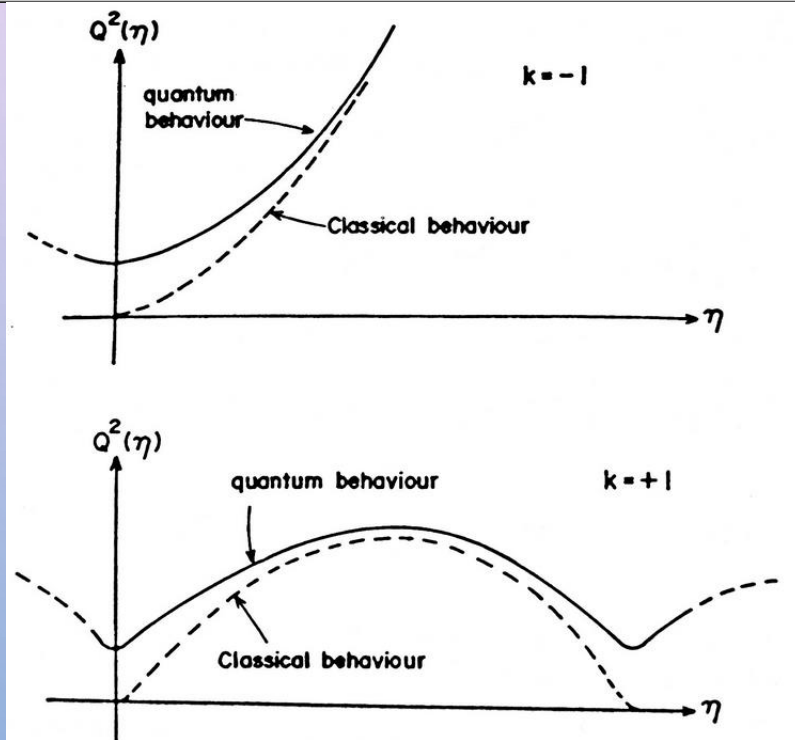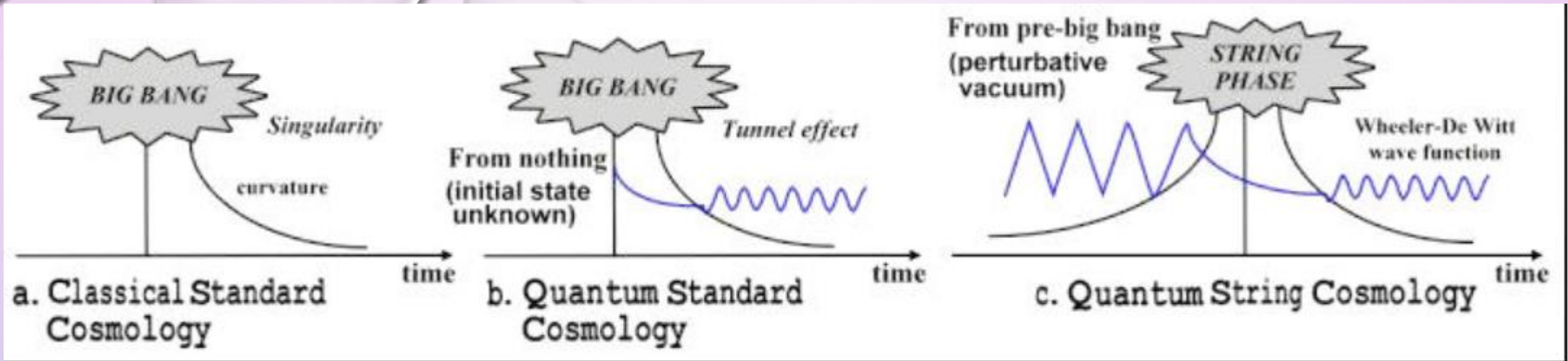
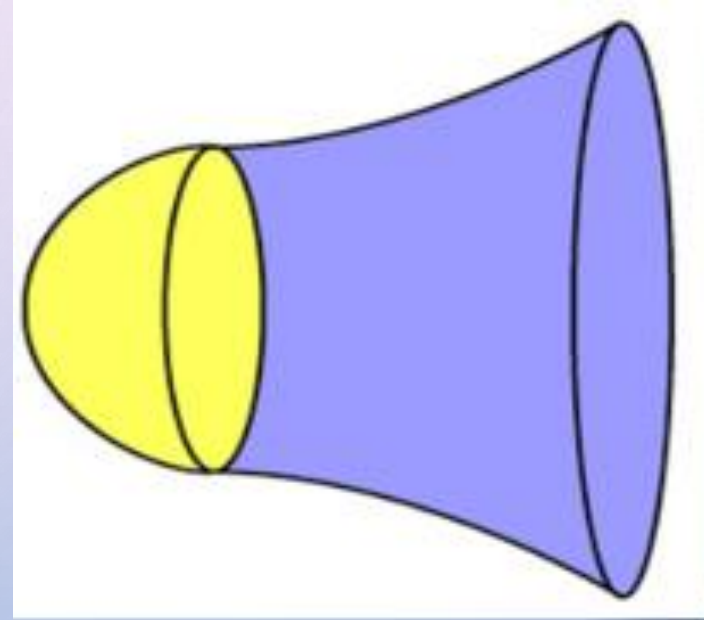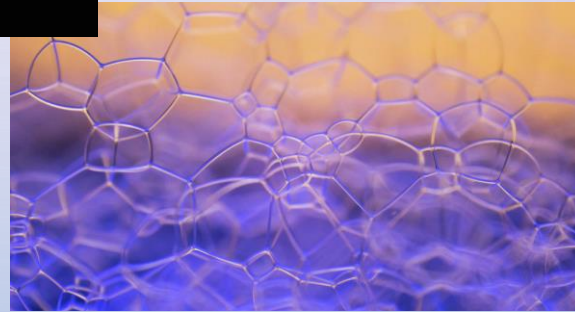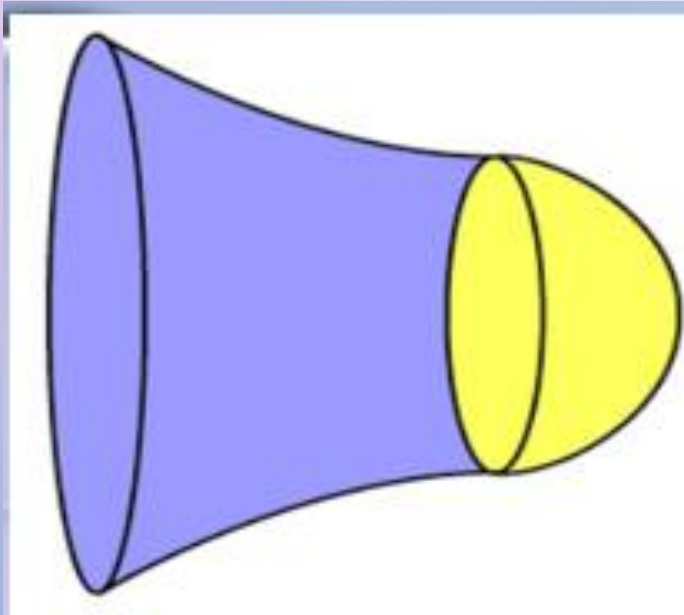$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\lambda = \frac{h}{p}$$

a. Classical Standard Cosmology

b. Quantum Standard Cosmology

c. Quantum String Cosmology

$$ds^2=-c^2dt^2+\cosh^2(Ht)[dx^2+dy^2+dz^2]$$

No Big Bang? Quantum equation predicts universe has no beginning

Quantum model predicts universe has NO beginning

2MASS Redshift Survey (2MRS)

The Universe's Quantum Energy Yin and Yang
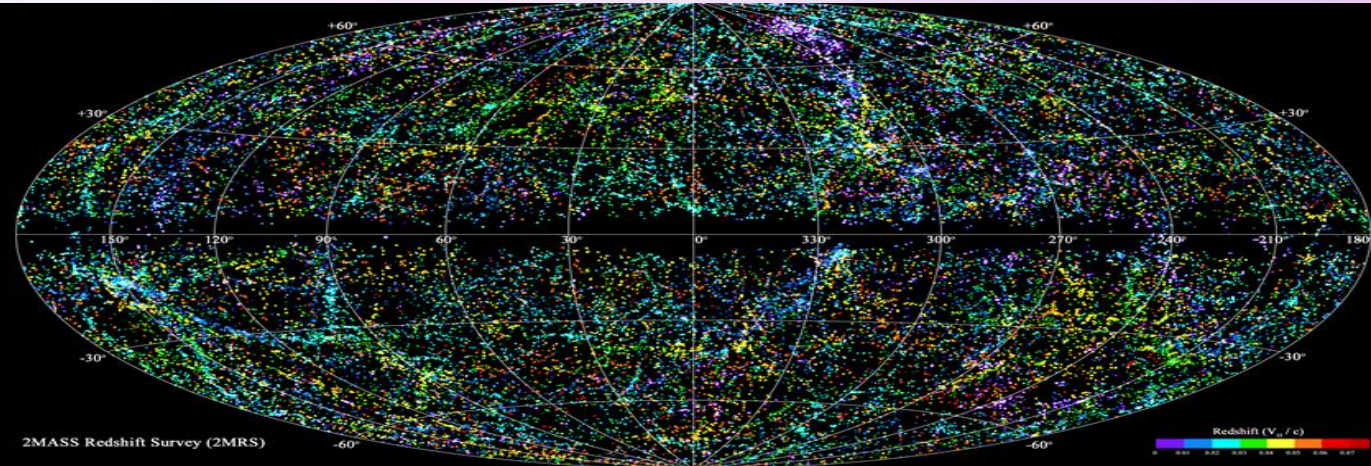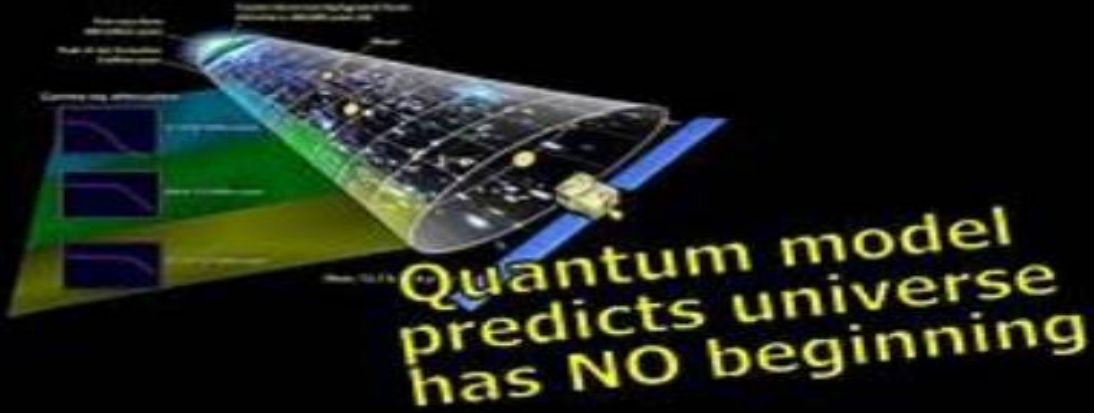
The physicists emphasize that their quantum correction terms are not applied ad hoc in an attempt to specifically eliminate the Big Bang singularity. Their work is based on ideas by the theoretical physicist David Bohm, who is also known for his contributions to the philosophy of physics. Starting in the 1950s, Bohm explored replacing classical geodesics (the shortest path between two points on a curved surface) with quantum trajectories.

یعنی جهان ما از یک تک مهبانگ اولیه اغاز نشده تا پایانی داشته باشد بلکه افت و خیزهای کوانتومی فضا زمان همواره مثل یک سوپ جوشان در حال تولید جهان های جدید و نابودی جهان های قبلی است و این فرایند با تونل زنی کوانتومی در امتداد زمان اتفاق می افتد

2. The gravity model

Let us start with the Brans-Dicke scalar-vector tensor gravity [10, 11]

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left\{ \phi R - \frac{\omega}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right\}$$

$$+ \frac{1}{16\pi} \int d^4x \sqrt{-g} \{ \zeta(x^\nu)(g^{\mu\nu} N_\mu N_\nu + 1) + 2\phi F_{\mu\nu} F^{\mu\nu} + U(\phi, N_\mu)$$

$$- \phi N_\mu N^\nu (2F^{\mu\lambda}\Omega_{\nu\lambda} + F^{\mu\lambda}F_{\nu\lambda} + \Omega^{\mu\lambda}\Omega_{\nu\lambda} - 2R^\mu_\nu + \frac{2\omega}{\phi^2}\nabla^\mu\phi\nabla_\nu\phi)\}, \tag{1}$$

where g is absolute value of determinant of the metric tensor $g_{\mu\nu}$ with signature as (-,+,+,+), ϕ is the Brans-Dicke scalar field and ω is the Brans-Dicke adjustable coupling constant. The tensor fields $F_{\mu\nu}$ and $\Omega_{\mu\nu}$ are defined versus the time like vector field N_μ as follows.

$$F_{\mu\nu} = 2(\nabla_\mu N_\nu - \nabla_\nu N_\mu), \qquad \Omega_{\mu\nu} = 2(\nabla_\mu N_\nu + \nabla_\nu N_\mu) \tag{2}$$

The above action is written in units $c = G = \hbar = 1$ and the undetermined Lagrange multiplier $\zeta(x^\nu)$ controls N_μ to be an unit time-like vector field. ϕ describes inverse of variable Newton's gravitational coupling parameter and its dimension is $(lenght)^{-2}$ in units $c = G = \hbar = 1$. Present limits of dimensionless BD parameter ω based on time-delay experiments [14, 15, 16, 17] requires $\omega \geq 4 \times 10^4$. Varying the above action functional with respect to ζ we obtain time-like condition on the vector field N_μ as follows.

$$g_{\mu\nu} N^\mu N^\nu = -1. \tag{3}$$

In the next section we apply the above mentioned action functional to study the anisotropic Bianchi I cosmological model.

3. Bianchi I quantum cosmology

Spatially homogenous but anisotropic dynamical flat universe is defined by the Bianchi I metric which from point of view of free falling (comoving) observer is defined by the following line element [18].

$$ds^2 = -dt^2 + e^{2a(t)}\{e^{-4b(t)}dx^2 + e^{2b(t)}(dy^2 + dz^2)\} \tag{4}$$

where x, y, z are cartesian coordinates of the comoving observer and t is cosmic time. In the above metric equation we assume that the spatial part has cylindrical symmetry for which $e^{a(t)}$ is an global isotropic scale factor and $b(t)$ represents a deviation from the isotropy. Substituting (4) into the equation (3) we obtain

$$N_\mu(t) = \begin{pmatrix} N_t \\ N_x \\ N_y \\ N_z \end{pmatrix} = \begin{pmatrix} \cosh \alpha \\ e^{a-2b}\sinh \alpha \cos \beta \\ e^{a+b}\sinh \alpha \sin \beta \cos \gamma \\ e^{a+b}\sinh \alpha \sin \beta \sin \gamma \end{pmatrix} \tag{5}$$

where (α, β, γ) are angular constant parameters of the vector field N_μ which makes as fixed its direction at the 4D anisotropic space time (4). Substituting (4) and (5) into the action functional (1) with some simple calculations, one can show that the action functional read

$$I = \frac{1}{16\pi} \int dx dy dz \int dt e^{3a}\left[-\omega\frac{\dot{\phi}^2}{\phi} - 6\phi\dot{a}^2 + 6\phi\dot{b}^2 - U(a,b,\phi)\right] \tag{6}$$

where dot $\cdot$ denotes to 'cosmic' time derivative $\frac{d}{dt}$ and we used the ansatz $\alpha = 0$ to eliminate frictional terms $\ddot{a}$ and $\ddot{b}$ in the action functional (6) (see ref.[5] for more detail). In general, if an action functional contains time derivative of velocity of the dynamical fields then there will be some frictional forces which cause that the extremum point of the action functional does not fixed. The latter kind of dynamical systems are not closed and so stable. They behave usually as chaotic dynamical systems. Hence we should eliminate these acceleration terms of the dynamical fields to fix extremum points of the system. Usually one obtained an effective action functional by integrating by part and removing divergence-less counterpart of the action functional. Here we can eliminate the frictional terms $\ddot{a}$ and $\ddot{b}$ without to use an effective action functional instead of the action functional (6) just by setting $\alpha = 0$. This restrict us to choose a particular direction for the time-like dynamical vector field (5) where the lagrangian of the system has not friction terms $\ddot{a}$ and $\ddot{b}$.

When we study canonical quantization of a mini-super-space quantum cosmology, then there the local coordinates do not have an important role but the dynamical fields themselves play an important role. Hence it will be useful to apply a conformal frame with the following conformal time $\tau(t)$ in what follows.

$$dt = F(t)d\tau, \qquad F = \frac{e^{3a}\phi}{16\pi} \tag{7}$$

where F is called as lapse (red shift) function in the ADM formalism of the decomposition of the background metric. Substituting (7) and

$$\phi = \frac{e^{Z(\tau)}}{G} \tag{8}$$

into the action functional (6) we obtain $I = \int dx dy dz \int \mathcal{L} d\tau$ where G is Newton's coupling constant and $\mathcal{L}$ is the Lagrangian density of the system from point of view of the conformal frame.

$$\mathcal{L} = -\omega Z'^2 - 6a'^2 + 6b'^2 - V(a, b, Z) \tag{9}$$

where $'$ denotes to derivative with respect to the conformal time τ and

$$V = \frac{F^2 U}{\phi} \tag{10}$$

is a suitable super-potential. Calculating canonical momenta of the fields (a, b, Z) as

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial a'}, \quad \Pi_b = \frac{\partial \mathcal{L}}{\partial b'}, \quad \Pi_Z = \frac{\partial \mathcal{L}}{\partial Z'} \tag{11}$$

and applying definition of the Hamiltonian density

$$\mathcal{H} = a' \Pi_a + b' \Pi_b + Z' \Pi_Z - \mathcal{L} \tag{12}$$

one can infer

$$\mathcal{H} = -\frac{\Pi_a^2}{24} + \frac{\Pi_b^2}{24} - \frac{\Pi_Z^2}{4\omega} + V(a, b, Z) \tag{13}$$

where we see that signature of the de Witt superspace metric is Lorentzian form $(-, +, -)$. In the next section we see that the canonical momentum operator of the anisotropic counterpart of the metric field b behaves as a time-evolution parameter of the system in the canonical quantum cosmology of the minisuperspace de Witt metric. However we obtain now Wheeler de Witt probability wave solution of the cosmological system under a quantization condition on the ADM energy of the system.

4. Canonical quantization

To study quantum stability of the Bianchi I cosmology we should first fix the potential $V(a, b, Z)$ where we choose anisotropic harmonic Oscillator potential defined on the mini-super-space de Witt metric as follows.

$$V(a, b, Z) = \frac{1}{2}K_a a^2 - \frac{1}{2}K_b b^2 + \frac{1}{2}K_Z Z^2. \tag{14}$$

Substituting the Dirac's canonical quantization condition for the momentum operators as

$$\hat{\Pi}_a = -i\frac{\delta}{\delta a}, \quad \hat{\Pi}_b = -i\frac{\delta}{\delta b}, \quad \hat{\Pi}_Z = -i\frac{\delta}{\delta Z}, \tag{15}$$

and the potential form (14) into the Hamiltonian density (13) we obtain the corresponding Wheeler de Witt wave equation $\hat{\mathcal{H}}\Psi(a, b, X) = 0$ as follows.

$$\left\{ \frac{1}{24}\frac{\delta^2}{\delta a^2} - \frac{1}{24}\frac{\delta^2}{\delta b^2} + \frac{1}{4\omega}\frac{\delta^2}{\delta Z^2} - \frac{K_a}{2}a^2 + \frac{K_b}{2}b^2 - \frac{K_Z}{2}Z^2 \right\}\Psi = 0. \tag{16}$$

Negativity sign of the differential operator $\frac{\delta^2}{\delta b^2}$ in the above equation shows that the anisotropy field Y can behave as the time evolution parameter in the three dimensional de Witt superspace metric. In other word the Kinetic terms in the above differential equation is similar to a Klein Gordon operator defined on the mini-super-space de Witt metric. To solve the equation (16) one can apply the standard method of separation of variables as follows. We assume the Wheeler de Witt wave solution to be separable as $\Psi(a, b, Z) = A(a)B(b)C(Z)$ and substitute it into the equation (16), then we can obtain the following differential equations for the fields $A(a), B(b), C(Z)$ respectively as follows.

$$\left[\frac{1}{24}\frac{\delta^2}{\delta a^2} + \epsilon_a - \frac{K_a}{2}a^2 \right]A(a) = 0 \tag{17}$$

$$\left[\frac{1}{24} \frac{\delta^2}{\delta b^2} + \epsilon_b - \frac{K_b}{2} b^2 \right] B(b) = 0 \tag{18}$$

$$\left[\frac{1}{4\omega} \frac{\delta^2}{\delta Z^2} + \epsilon_Z - \frac{K_Z}{2} Z^2 \right] C(Z) = 0 \tag{19}$$

where the constants of separation of variables $\epsilon_{a,b,Z}$ satisfy the following relation.

$$\epsilon_a + \epsilon_Z = \epsilon_b. \tag{20}$$

One can show that the equations (17), (18) and (19) can be described by the well known Hermit polynomials if we set the following quantization conditions on the parameters $\epsilon_{X,Y,Z}$.

$$\epsilon_a = \left(N_a + \frac{1}{2} \right) \sqrt{\frac{K_a}{12}}, \quad \epsilon_b = \left(N_b + \frac{1}{2} \right) \sqrt{\frac{K_b}{12}}, \quad \epsilon_Z = \left(N_Z + \frac{1}{2} \right) \sqrt{\frac{K_Z}{2\omega}} \tag{21}$$

in which $N_{\{a,b,Z\}}$ can take quantized values $0, 1, 2, 3, \cdots$. In the latter case we will obtain normalized form of the solutions $A(a)$, $B(b)$ and $C(Z)$ as follows.

$$A_{N_a}(a) = \frac{H_{N_a}(\lambda_a a) e^{-\lambda_a a^2 / 2}}{\sqrt{2^{N_a} N_a! \sqrt{\pi}}}, \quad \lambda_a = (12 K_X)^{\frac{1}{4}} \tag{22}$$

$$B_{N_b}(b) = \frac{H_{N_b}(\lambda_b b) e^{-\lambda_b b^2 / 2}}{\sqrt{2^{N_b} N_b! \sqrt{\pi}}}, \quad \lambda_b = (12 K_b)^{\frac{1}{4}} \tag{23}$$

$$C_{N_Z}(Z) = \frac{H_{N_Z}(\lambda_Z Z) e^{-\lambda_Z Z^2 / 2}}{\sqrt{2^{N_Z} N_Z! \sqrt{\pi}}}, \quad \lambda_Z = (2\omega K_Z)^{\frac{1}{4}} \tag{24}$$

in which $H_{N_a}, H_{N_b}, H_{N_Z}$ are Hermit polynomials. Multiplying the above solutions we obtain $\Psi_{N_a N_b N_Z} = A_{N_a}(a)B_{N_b}(b)C_{N_Z}(Z)$ which in fact describes quantum fluctuations of the metric field of the Bianchi I cosmology defined by (4) when the cosmological system is in quantum state (N_a, N_b, N_Z) with corresponding eigne energy $(\epsilon_{N_a}, \epsilon_{N_b}, \epsilon_{N_Z})$. Now we can show that general solution of the Wheeler de Witt wave equation (16) by expanding it versus the eigne functionals such that

$$\Psi(X,Y,X) = \Sigma_{N_a=0}^{\infty}\Sigma_{N_b=0}^{\infty}\Sigma_{N_Z=0}^{\infty} D_{N_a N_b N_Z}\Psi_{N_a N_b N_Z}(a,b,Z) \tag{25}$$

where the coefficient $D_{N_X N_Y N_Z}$ describes probability of the quantum Bianchi I cosmology which takes the eigne state $\Psi_{N_X N_Y N_Z}$. This can be determined by regarding the initial condition of the system and orthogonal condition on the Hermit polynomials. However no one does not know about initial condition of "physical cosmology" but we can predicts some physical statements about our obtained solutions in what follows. Substituting (21) into the condition (20) we can obtain allowable eigne states of the system as follows.

$$\left(N_a + \frac{1}{2}\right)\sqrt{\frac{K_a}{K_b}} + \left(N_Z + \frac{1}{2}\right)\sqrt{\frac{6}{\omega}}\sqrt{\frac{K_Z}{K_b}} = \left(N_b + \frac{1}{2}\right). \tag{26}$$

5. ADM energy and mass

In the previous section we assume the ADM mass of the cosmological system has a zero value. If it is not permissible then we should solve the extended Wheeler de Witt wave equation as $\hat{\mathcal{H}}\Psi = \mathcal{M}\Psi$ in which $\mathcal{M}$ is called to be the ADM mass of the system. In the latter case the equation (20) should be extended to the following form.

$$\mathcal{M} = \epsilon_b - (\epsilon_a + \epsilon_Z) \tag{27}$$

which by substituting the quantization conditions (21) we obtain a quantization condition on the ADM mass of the Bianchi I cosmological model such that

$$\mathcal{M}_{N_a N_b N_Z} = \left(N_b + \frac{1}{2}\right)\sqrt{\frac{K_b}{12}} - \left(N_a + \frac{1}{2}\right)\sqrt{\frac{K_a}{12}} - \left(N_Z + \frac{1}{2}\right)\sqrt{\frac{K_Z}{2\omega}} \tag{28}$$

In fact ADM energy is a special way to define the energy in general relativity, which is only applicable to some special geometries of spacetime that asymptotically approach a well-defined metric tensor at infinity. If the background metric approaches to Minkowski space asymptotically then the Neother's theorem implies that the ADM energy or mass should be invariant because of time independence of the Minkowski metric. According to general relativity, the conservation law for the total energy does not hold in more general. For instance for time-dependent background metrics it will be violated. For example, it is completely violated in physical cosmology. In fact cosmic inflation in particular is able to produce energy or mass from "nothing" because the vacuum energy density is roughly constant, but the volume of the Universe grows exponentially. Here we can modeled the violation of the ADM energy in the anisotropic Bianchi I inflationary cosmology by quantization approach. Here we show that the raising the ADM energy in the Bianchi I cosmology can be described by quantum fluctuations of the mass parameters of the fields (a, b, Z) which we called with K_a, K_b, K_Z respectively.

6. Big Bang naked singularity

The background metric (4) shows that the naked singularity lies on the particular hypersurface $(a, b) \to (-\infty, -\infty)$ at the classical cosmological feature where the big bang originated. Applying $(a, b) \to (-\infty, -\infty)$ one can infer that for naked singularity we have $A(-\infty) = 0 = B(-\infty)$. This reads

$$\lim_{(a,b)\to(-\infty,-\infty)} \Psi(a, b, Z) = 0. \tag{29}$$

Physical interpretation of the above result can be described as follows: There is a zero probability where the big bang originates from a naked singularity. In other words the big bang originates from an anisotropic quantum Oscillation of the quantum matter fields (a, b, Z) at small scales of the space time. This happened because of the uncertainty relations between the fields and the corresponding canonical momenta as $\Delta X_i \Delta \Pi_{X_i} \equiv 1$ where $X_i = (a, b, Z)$.

7. Concluding remarks

In this paper we choose scalar vector tensor Brans Dicke gravity to study anisotropic Bianchi I cosmology in the canonical quantization approach. We solved Wheeler de Witt wave equation and obtained its solutions versus the Hermit polynomials for an anisotropic harmonic Oscillator potential. We obtained a quantization condition on the ADM boundary energy of the system. Mathematical calculations predict that the inflation of the universe can be originated from a big bang state without a naked singularity. Furthermore anisotropic counterpart of the metric field can play as a time evolution parameter on the mini-super-space because of Lorentzian signature of the de Witt metric. As a future work one can study backreaction of the Hawking thermal

radiation of the quantum matter (the vector and the Brans Dicke scalar) fields on the solutions of the Wheeler de Witt equations and so the corresponding eigne energies (quantized ADM energy) of the system which does not considered here.

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با سپاس از توجه سبزتان